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Strategic Asset Valuation and Higher Stochastic Moments: An Adjusted Black-Scholes Model

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Abstract: Strategic asset valuation is a complex problem which influences the decision making in companies, such as decisions to differing or selling a project. Uncertainty takes over the manager when defining the attributes of the density function representing values that could assume the asset in the future. In this paper, we include not only its mean and variance, but also stochastic higher moments of this function (asymmetry and kurtosis). This paper is original since we prove how strategic decisions are subject to the impact of higher moments in the expanded value of assets. This is why we include a detailed sensitivity analysis to clarify changes in valuation because of the influence of asymmetry (ε) or kurtosis (κ) on the underlying asset distribution. Hence, we obtained theoretical solutions to asset valuations that would have been impossible to solve.

Keywords: Strategic Asset; Asymmetry; Kurtosis; Edgeworth Expansion; Continuous Time; Real Option; Firm Valuation; Black-Scholes Model

JEL Classifications: G32, M13, G13

1. Introduction

Real asset valuation is a complex problem that influences strategic decision making on firms such as differing or selling a project. Uncertainty takes over the entrepreneur while defining density function that represents different values that will assume the asset in the future. On this situation, we do not only include its mean and variance, but also stochastic higher moments.

Milanesi, Pesce & El Alabi (2013) presents a solution in discrete time. This article offers a solution to the mentioned problem in continuous time working on a case study. Therefore, this work's objective is to propose a technique to value strategic assets using the classic option valuation model in continuous time (Black & Scholes, 1973) together with the Edgeworth expansion in order to incorporate stochastic higher moments on the underlying distribution. Thus, we propose to adapt the normal function and to test the model over the case study.

We structure the paper on the following manner: section 2 presents theoretical background where we describe meanly what the Edgeworth expansion is and how it is adjusted to Black and Scholes (BS). In section 3, we work with the Black-Scholes-Edgeworth (BSE) model applied as a case study. From this point, we firstly estimate the implicit volatility curve, and then we value different real options (differing, selling, and complex combined strategies). Lastly, we conclude on the importance of utilizing this type of models to include extreme events in future scenarios of real asset strategic values.

2. Theoretical Background

2.1 Edgeworth expansion

Jarrow and Rudd (1982) applied Edgeworth expansion on Schleher technique (1977) where the real probability distribution $Z(x)$ is approach by a different one called $G(x)$. In statistics, this technique is known as the Edgeworth expansion (Cramer, 1946; Kendall & Stuarts, 1977). The expansion approaches a more complex probability distribution to a simpler alternative such as the normal or lognormal distribution. This technique allows the expansion's coefficient to depend on the moments, either the original distribution or the approach one. Therefore, we obtained theoretical solutions to asset valuations that would have been impossible to solve. From Jarrow and Rudd (1982), and Baliero Filho and Rosenfeld (2004) as well, we contrast this methodology in order to explain volatility smile[1].

Following Baliero Filho and Rosenfeld (2004), we develop the expansion. Assume a series of independent, identically distributed random variables (iid) $x_1, x_2, ..., x_n$ with mean μ and finite variance σ^2. On this case, the random variable is defined as (1):

$$X_n = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (1)$$

Probability distribution of the random variable is obtained through an expansion on the characteristic function distribution $x_n(t) = E(e^{itS_n})$ resulting in $e^{-t^2/2}$ for the normal distribution, and where S_n represents the underlying value at moment *n*. The characteristic function is expanded the following manner (2):

$$x_n(t) = exp\left[-\frac{t^2}{2} + \frac{1}{n^{\frac{1}{2}}}\kappa_3(it^3) + \cdots + \frac{1}{n^{\frac{j-2}{2}}}\frac{1}{j!}\kappa_j\left(it^j\right)\right] \qquad (2)$$

Values κ indicate stochastic moments on the underlying distribution S_n. Here, first moment is equal to $E(S_n) = \kappa_1 = 0$ and second moment is equal to $Var(S_n) = \kappa_2 = 1$. Baliero Filho and Rosenfeld (2004) come up to the Edgeworth expansion (3):

$$g(x) = \left(1 + \frac{\varepsilon}{6}(x^3 - 3x) + \frac{k-3}{24}(x^4 - 6x^2 + 3) + \frac{\varepsilon^2}{72}(x^6 - 15x^4 + 45x^2 - 15)\right)z(x) \quad (3)$$

This expression is valid until the order *1/n*, asymmetry is defined as $\varepsilon = \kappa_3$ and kurtosis $\kappa = \kappa_4 + 3$, incorporating factors *1/n* on this parameters. Function *g(x)* is the product between

[1] It is an implicit volatility patron detected in numerous works (Rubinstein, 1994). It suggests that the Black and Scholes option valuation model tends to undervalue options that are way in- *or* way out-of-the-money.

Gaussian distribution N(0,1) *z(x)* and the expression belongs to the expansion.

2.2 Black and Scholes model and the adjustment with the Edgeworth expansion

Financial and real assets returns' distributions hardly ever adjust to the classic normal behavior having asymmetry and weight on the extremes. New projects, technological developments, and market innovation are characterized by the lack of comparable assets and the absence of price and returns observations. Assuming the underlying stochastic process over the first two moments (mean-variance) could generate errors in evaluating the real asset or the underlying financial asset. Therefore, it is necessary to incorporate stochastic higher moments allowing a better valuation and volatility estimation[2].

Baliero and Rosenfeld (2004) model derivation starts from the asset growth rate defined as:

$$\mu T = rT - log\left(1 + \frac{\kappa-3}{24}\left(\sigma\sqrt{t}\right)^4 + \frac{\varepsilon}{6}\left(\sigma\sqrt{t}\right)^3 + \frac{\varepsilon}{72}\left(\sigma\sqrt{t}\right)^6\right) \tag{4}$$

In this equation, *r* is the risk free rate, *T* is the time left until the option expires, σ is the underlying asset volatility, ε is asymmetry, and κ is kurtosis. Having asymmetry $\varepsilon=0$ and kurtosis $\kappa=3$ (normal), then $\mu=r$. Thus, we obtain same solution as BS. Conventional expression of the BS model for call options is:

$$C_0^{BS} = S_0 N(d_1) - Ke^{-rt}N(d_2) \tag{5}$$

where C_0^{BS} is the option theoretical value, V_0 is the underlying asset present value, $N(d_i)$ is the cumulative normal distribution of the variable d_i, K is the strike price, *r* is the risk free rate, and *t* the expiration date[3]. Variables d_1 and d_2 are estimated in the following manner: $d_1 = [ln(V_0/K) + (r + \sigma^2/2)T]/\sigma\sqrt{T}$ and $d_2 = d_1 - \sigma\sqrt{T}$.

For the general case, the expression that determines the option expected value is:

$$C_0^{Edge} = e^{-rt}\int_{-\infty}^{\infty} dxg(x)Max(\,S_0 e^{\left(\mu-\frac{\sigma^2}{2}\right)T+\sigma\sqrt{Tx}} - K, 0) \tag{6}$$

where C_0^{Edge} is the call theoretical value, *r* is the risk free rate, *T* is the time horizon until expiration, S_0 the underlying asset market value on $t=0$, σ is volatility, *K* is the strike price, and *g(x)* is the transformed function. The integral could be converted into a closed solution model for the option valuation (Baliero Filho & Rosenfeld, 2004) resulting in a two-section divided equation: the BS model and the Edgeworth expansion:

$$\boldsymbol{C_0^{Edge} = C_0^{BS} + \left(\frac{e^{u-rT-x_m^2/2+\sigma\sqrt{T}x_m}}{72\sqrt{2\pi}} V_0\left((\sigma\sqrt{T})^5\varepsilon^2 + (\sigma\sqrt{T})^4\varepsilon^2 x_m + \left(\sigma\sqrt{T}\right)^3\left(3(\kappa-3)\right) + \right.\right.}$$

[2] Stochastic moments in financial derivatives could be inferred from market prices. This allows to an adjusted volatility measure. In valuation models in real options, moments could be sensitize presenting a range of values related to the strategic flexibility valued.

[3] The expression $V_0N(d_1)$ is the expected present value related to the underlying asset in case that the option ends in-the-money, being $N(d_1)$ the risk adjusted probability that the underlying ends above the exercise price at expiration. $Xe^{-rt}N(d_2)$ is the expected present value of the exercise price if the options ends up in-the-money, being $N(d_2)$ the risk adjusted probability that the option being exercised. (Carmichael, Hersh & Parasu, 2011).

$$\varepsilon^2(x_m^2-1)+\left(\sigma\sqrt{T}\right)^2\left(12\varepsilon-3(\kappa-3)x_m+\varepsilon^2x_m(x_m^2-3)\right)+\left(\sigma\sqrt{T}\right)(12\varepsilon x_m+3(\kappa-3)(x_m^2-1)+\varepsilon^2(x_m^4-6x_m^2+3)\Big)\Bigg)+\left(\frac{e^{-rT-\frac{x_m^2}{2}}}{72\sqrt{2\pi}}\left(e^{u+\sigma\sqrt{T}x_m}V_0-K\right)(3(\kappa-3)x_m(x_m^2-3)+12\varepsilon(x_m^2-1)+\varepsilon^2x_m(x_m^4-10x_m^2+15))\right)+\left(\frac{e^{u-rT-\frac{\sigma^2T}{2}}}{72}V_0\,N(d_1)\left(\left(\sigma\sqrt{T}\right)^4 3(\kappa-3)+\left(\sigma\sqrt{T}\right)^6\varepsilon^2+12\left(\sigma\sqrt{T}\right)^3\varepsilon\right)\right) \quad (7)$$

In the previous equation, C_0^{BS} is the call option value according to BS, r is the risk free rate, T is the time horizon until expiration, S_0 is the underlying asset market value in $t=0$, σ is volatility, K is the strike price, u is the asset growth rate (equation 4), ε is asymmetry, κ is kurtosis, and $x_m=\frac{\log\left(\frac{K}{S_0}\right)-\left(u-\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$ is the minimum value to guarantee that the integer from equation (6) be positive. Variable x_m is the same as d_1 in the BS model with $\varepsilon=0$ and $\kappa=3$. In cases like this (normality), the transformed model converges to BS. Same criterion follows the put option. To the BS equation $P_0^{BS}=V_o\left(N\left(d_1\right)-1\right)-Ke^{-rT}\left(1-N\left(d_2\right)\right)$, we add the Edgeworth expansion $g(x)$, $P_0^{edge}=P_0^{BS}+g(x)$. We come up to the same result applying the put-call parity.

3. The Black-Scholes-Edgeworth (BSE) Model: An Application Case

3.1 Estimating the Implicit Volatility Curve

In order to illustrated how stochastic higher moments impact on the implicit utility curve, this utility curve will be derived using equation (7) through an iterative process[4]. On this process, the equation is equaled to the observed market price (C_t)[5] to get implicit values related to deviation (σ), asymmetry (ε), kurtosis (κ). Thus, we establish the following restrictions[6]: $C_t \geq 0$; $\sigma \geq 0$; $-0{,}8 \leq \varepsilon \leq 0{,}8$; $3 \leq \kappa \leq 5{,}4$. The process starts establishing higher moments with value zero and volatility with

[4] The iterative process is solved using Solver tool from Microsoft Excel ®. We define each cell where we introduce the market price from equation (7) as an objective value. This is equal to the prime market value. Volatility, asymmetry, kurtosis, and risk free rate are cells to be changed.

[5] On every model where variables are estimated implicitly, it is assumed that market values are correct.

[6] Restrictions related to asymmetry and kurtosis, are defined according to the potential null values for the function (Baliero Filho & Rosenfeld, 2004). These restrictions are defined by Solver in Microsoft Excel ®.

its implicit value[7]. Once we get the implicit values for the stochastic moments, we proceed to obtain implicit volatility from the classic BS equation. To do this, we set again higher moments as $\varepsilon=0$; $\kappa=3$.

We valued Facebook Inc. (FB) vanilla options listed in the National Association of Securities Dealers Automated Quotation (NASDAQ). In order to use the variable estimation on a hypothetical case of real option valuation, we selected contracts with expiration date on January, 15th, 2016 and different exercise prices. Risk free rate belongs to a treasury bill with expiration in a year from t=0 being 0.16% annually[8]. Values on in-the-money option contracts are taken from Yahoo Finance[9] which expire on February, 6th, 2016. On Annex 1, we expose data related to the valued contract. The following table compares implicit volatility taken from BS and BSE models. First column are strike prices for our contracts. Second and third columns are implicit volatility from BS and BSE models. Fourth and fifth columns are asymmetry and kurtosis implicit on BSE model. Sixth column is the portion of price related to the BS model (having $\varepsilon=0$, $\kappa=3$ and BSE volatility). Seventh column is the magnitude of price explained by the expansion. Eighth column are market prices. Finally, ninth column is the percentage that the expansion represents on price.

Table 1. Implicit values are obtained from the iterative process related to equation (7)

Strike	σ (i) BS	σ (i) BSE	$\boldsymbol{\varepsilon}$	$\boldsymbol{\kappa}$	BS	E	Price	E/Price
13.00	86.51%	83.60%	0.0011	3.00	62.72	0.03	62.75	0.05%
15.00	126.87%	79.22%	0.0558	3.15	60.75	1.45	62.20	2.33%
18.00	73.25%	72.96%	-0.0383	2.91	57.79	-0.79	57.00	-1.39%
20.00	88.28%	68.36%	0.0368	3.08	55.81	0.64	56.45	1.14%
23.00	49.76%	63.24%	-0.0142	2.97	52.85	-0.20	52.65	-0.38%
25.00	83.14%	61.92%	0.0751	3.13	50.93	0.97	51.90	1.86%
30.00	65.28%	55.29%	0.0543	3.07	46.05	0.50	46.55	1.08%
33.00	83.05%	55.47%	0.2918	3.29	43.32	2.31	45.63	5.06%
35.00	45.74%	52.68%	-0.0496	2.99	41.36	-0.36	41.00	-0.89%
38.00	71.05%	57.06%	0.1981	2.95	39.11	1.49	40.60	3.67%
40.00	42.15%	46.91%	-0.0585	3.04	36.53	-0.33	36.20	-0.92%
43.00	47.40%	46.54%	0.0152	2.98	33.91	0.09	34.00	0.25%
45.00	47.44%	45.36%	0.0420	2.93	32.11	0.24	32.35	0.73%
47.00	31.39%	36.52%	-0.0948	3.20	29.51	-0.41	29.10	-1.41%
50.00	38.90%	39.27%	-0.0081	3.03	27.28	-0.05	27.23	-0.17%
52.50	37.33%	37.30%	0.0006	3.00	25.00	0.00	25.00	0.02%
55.00	36.66%	36.68%	-0.0003	3.00	22.95	0.00	22.95	-0.01%
57.50	35.62%	35.79%	-0.0028	3.03	20.93	-0.03	20.90	-0.15%
60.00	35.02%	35.10%	0.0000	3.02	19.02	-0.02	19.00	-0.08%

[7] We obtain implicit volatility using Microsoft Excel ® where we define the BS cell with the market value to change volatility. Market prices are taken from http://finance.yahoo.com/q/op?s=FB&date=1452816000.

[8] We obtained data from the Federal Reserve website: http://www.federalreserve.gov/releases/h15 cuadro h.15 interest rate on 2/2015.

[9] Yahoo Finance website: http://finance.yahoo.com/q/op?s=FB&date=1452816000

62.50	34.50%	34.52%	-0.0002	3.01	17.21	-0.01	17.20	-0.04%
65.00	33.19%	33.23%	-0.0001	3.01	15.31	-0.01	15.30	-0.06%
67.50	31.81%	31.98%	0.0001	3.01	13.46	-0.01	13.45	-0.05%
70.00	32.38%	32.37%	0.0000	3.00	12.20	0.00	12.20	0.00%
72.50	31.89%	31.89%	0.0000	3.00	10.78	0.00	10.78	0.00%
75.00	31.36%	31.36%	0.0001	3.00	9.45	0.00	9.45	0.00%
77.50	30.86%	30.86%	-0.0002	3.00	8.23	0.00	8.23	0.00%
80.00	30.35%	30.35%	-0.0005	3.00	7.11	0.00	7.11	-0.01%
82.50	30.39%	30.38%	0.0006	3.00	6.25	0.00	6.25	0.01%
85.00	29.61%	29.61%	-0.0010	3.00	5.25	0.00	5.25	-0.03%
87.50	29.92%	29.91%	0.0021	3.00	4.65	0.00	4.65	0.08%
90.00	29.39%	29.39%	-0.0001	3.00	3.90	0.00	3.90	0.00%
95.00	28.82%	28.83%	-0.0008	3.00	2.77	0.00	2.77	-0.13%
100.00	28.57%	28.61%	-0.0007	3.01	1.99	-0.01	1.98	-0.34%
105.00	28.61%	28.65%	0.0000	3.01	1.45	-0.01	1.44	-0.45%
110.00	29.26%	29.13%	-0.0020	2.98	1.11	0.02	1.13	1.62%
115.00	28.85%	28.84%	-0.0001	3.00	0.77	0.00	0.77	0.07%
120.00	28.40%	28.56%	0.0061	3.01	0.53	-0.02	0.51	-2.97%
125.00	28.80%	28.86%	0.0023	3.00	0.40	-0.01	0.39	-1.33%
130.00	29.01%	29.10%	0.0022	3.00	0.30	-0.01	0.29	-1.93%
135.00	29.71%	29.39%	-0.0053	3.01	0.22	0.02	0.24	6.35%
140.00	29.61%	29.59%	-0.0002	3.00	0.17	0.00	0.17	0.47%
145.00	32.99%	30.85%	-0.0262	3.05	0.16	0.10	0.26	36.61%
150.00	30.10%	31.03%	0.0066	2.99	0.13	2.99	3.11	95.93%
155.00	32.36%	31.69%	-0.0045	3.01	0.11	0.02	0.13	15.08%

Data source: Own elaboration

In contrast to other works where out-of-the-money contracts are selected (Milanesi, 2014), and asymmetry and kurtosis are found on market prices of these contracts, this paper selects in-the-money contracts. We could observe differences between σ (BSE) y σ (BS) implicit volatility as the weight presented by asymmetry ε and kurtosis κ. We would highlight the last strike prices (K=$145, $150, $155) where the expansion participation over price rises to 36.61%, 95.93%, and 15.08%. As the option goes out-of-the-money, theoretical value is explained mainly by higher moments, especially kurtosis. We could argue that out-of-the-money options value behavior emerges from extremes values of the analyzed underlying asset.

We must mention that implicit volatility obtained through the BS model assumes lognormal behavior for the underlying asset. BSE model offers a better measure for volatility since it sets apart higher moments. The volatility curve that we get from the BS model has a smile shape because of jumps of out-of-the-money contracts. However, volatility related to the underlying asset should be just one since the asset is unique. Apart from the type of contract, it should be stable. The fact that the option contract has value, independently that is way out-of-the-money, obeys the existence of higher moments. In particular, kurtosis which explains fat tails or extreme events occurrence probabilities. Next figure compares implicit volatility for the FB stock with BS and BSE models according to table 1 above.

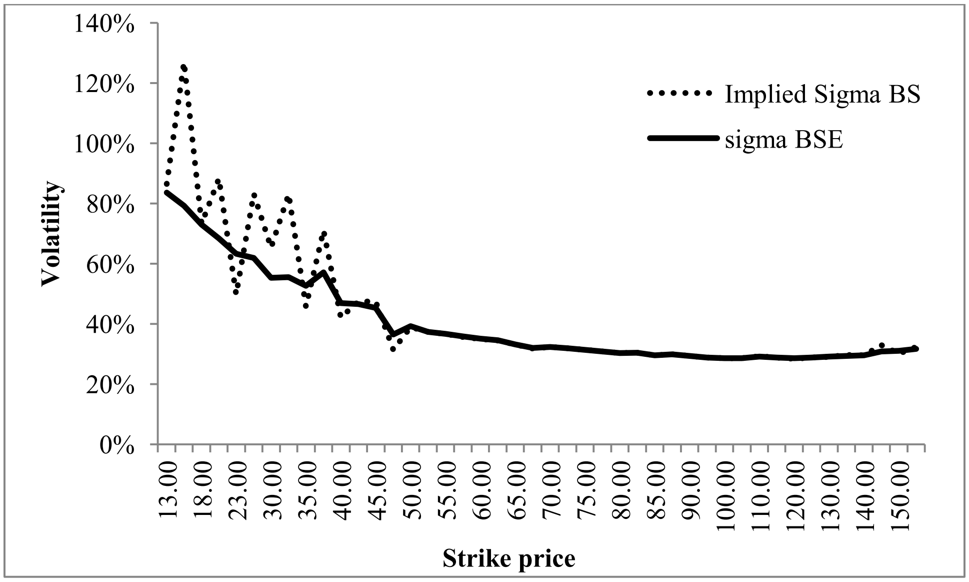

Figure 1. Implicit volatility for BS and BSE derivate from call option contracts for FB (own elaboration)

We could appreciate a higher flattening on the curve estimated with the BSE related to the traditional model. BSE separates higher moments of implicit volatility and, as a consequence, the softer behavior of the implicit volatility curve. We must consider this while applying real options in valuing strategic flexibility on investment projects.

3.2 Real Options and the BSE Model: Analysis of differing, Selling, and Combined Options

In order to illustrate differences while valuating options between BS and BSE models, we assume that the same firm is planning on developing a new app to be commercialized. The project has two stages: the pilot stage and the commercial stage where the beginning of the commercial stage is conditioned by the final results of the pilot stage. First phase lasts five years (t=5). On the second phase, we estimate the present value of the benefits, $E(FV_5) = \$4,375$ (thousands) with a $1,345 (thousands) deviation σ_5 through a series of scenarios. The firm's cost of capital (WACC) is assumed on 10.5% and the risk free rate is 5.5% annually. The investment needed for the second phase (commercialization) to be made on the fifth year is $5,000 (thousands), risk free.

If the project is valued by the traditional net present value (NPV) approach, we get: $PV(FV_5; k) =$ $2,588.05 (thousands); $PV(I_5; r) =$ 3,797.86 (thousands). Therefore, $NPV = PV(FV_5; k) - PV(I_5; r)$ = -$1,209.81 (thousands). The obtained result drives us to reject the project since efforts in research and development (R&D) made during the first stage will not result in favorable outcomes during the second stage. Consequently, we will not initiate the pilot stage. However, it does not consider the added value for having flexibility during the project since it assumes that the investment is irreversible and inflexible. On this case, we assume that compromise of investing on the second stage t=5 is assumed to be on t=0. Thus, strategic flexibility must be quantified using real option valuation models. Options contained in the project are: (a) to differ the investment until moment t=5 waiting for more information related to the market evolution once introducing this new app; (b) to develop the project and investing and then, on t=5, selling the project in $2,500 (thousands) if the non-favorable evolution occurs; (c) to combine the option of differ and then investment or selling the project. The first alternative is similar to a call, the second to a put, and the third to a strangle which is a strategy that unifies investment (buying) with the possibility of abandoning (selling).

The objective is to determine strategic flexibility value with expanded real option models. First, expanded value (EV) is equal to the traditional value (NPV) plus the value of the real options (RO):

$$EV = NPV + RO \tag{8}$$

On this case, values related to differing, selling, and the combination are determined by the following parameters: underlying present value $V_0 = E(FV_5) \times e^{-kt} = \$4375.e^{-0.105\times 5} = \$2{,}588.05$; strike price for the differing option $K = \$5{,}000$; selling option $X = \$2{,}500$; risk free rate $r = 5.50\%$; and time until exercise for both options $t = 5$. Volatility expressed as a percentage is obtained by clearing[10] from the expression $SD = V_0 e^{kt}\sqrt{e^{\sigma t} - 1} = \$1345 = \$2{,}588.05 e^{0.105\times 5}\sqrt{e^{\sigma 5} - 1}$ (Wilmott, Howison & Dewynne, 1995) where $\sigma = 13.44\%$.

The value of the differing option according to BS (ε =0; κ=3) comes up from the following expression: $C_0 = 2588.05\, N(d_1) - \$5000 e^{-0.055t} N(d_2)$ with these parameters: $d_1 = -1.1259$; $d_2 = -1.4264$; $N(d_1) = 0.13009$; $N(d_2) = 0.07686$. The strategic value is \$44.76 (thousands), thus, differing option value goes up to \$1,254.57 (thousands) indicating the convenience to invest in phase 1, instead of not investing, while waiting to new information on t=5 to complete phase 2.

In the selling option, we use the expression of the put option on BS: (ε=0; κ=3); $P_0 = 2{,}500 e^{-0.055t} N(-d_2) - 2{,}588.05 N(-d_1)$ with parameters:

$d_1 = 1.1805$; $d_2 = 0.8799$; $N(-d_1) = 0.1189$; $N(-d_2) = 0.1894$. The strategic value is \$52.01 (thousands), the abandoning or selling option goes up to \$1,261.81 (thousands).

Finally, the combined strategy offers the project an EV of \$96.67 (thousands) which is the sum of strategic values related to buying and selling options. The EV is \$1,306.58 (thousands) which is the feasibility of making R&D on the first stage and then, on t=5, making the investment and commercializing or, contrarily, transferring the app license. If the previsions actually occur, selling the license is more profitable than commercializing the app.

As we previously mentioned, this type of projects hardly ever present a lognormal behavior. In fact, its success probability depends on extreme events. The convenience of transferring the license or investing and commercializing will depend on the stochastic behavior of the underlying asset. Therefore, we must incorporate stochastic higher moments.

We used the defined equations in section 2 in order to value differing and selling options contained in the project considering stochastic higher moments. Thus, while analyzing potential values for the options, we proceed to sensitize higher moments: asymmetry ε=[-0,7; 0,7] and kurtosis κ=[3; 5,4]. Volatility remained fixed. On the following tables, we expose values related to the project strategic value with the differing option (table 2), the selling option (table 3), and the combined differing and selling option (table 4).

On these tables, we highlight project strategic values in cases where we assume normality. The higher the positive asymmetry and kurtosis (meso-kurtosis), the value of the differing and abandoning options goes up. On the case of negative asymmetries, they counteract the higher value that is obtained by the fourth stochastic moment. Tables focus on the impact of kurtosis on the differing value (table 2), if it is compared to selling option (table 3). In case the project allows us to

[10] Volatility is obtained by iterating the expression with the search objective function from MS Excel ®.

implement both strategies concomitantly, values are exposed on table 4. These values come up from summing up values on tables 2 and 3.

Table 2. Sensitivity of differing option value according on asymmetry and kurtosis (Own elaboration)

ε \ κ	3.0	3.3	3.5	3.8	4.0	4.3	4.5	4.8	5.0	5.3	5.5
0.7	76.61	79.47	81.37	84.21	86.11	88.95	90.84	93.68	95.57	98.40	100.2
0.6	71.08	73.93	75.84	78.69	80.59	83.44	85.34	88.18	90.07	92.91	94.80
0.5	65.87	68.73	70.64	73.50	75.40	78.26	80.16	83.01	84.90	87.74	89.64
0.4	60.99	63.86	65.77	68.63	70.54	73.40	75.30	78.16	80.06	82.91	84.81
0.3	56.43	59.31	61.23	64.10	66.01	68.87	70.78	73.64	75.55	78.40	80.30
0.2	52.21	55.09	57.01	59.89	61.80	64.68	66.59	69.45	71.36	74.22	76.13
0.1	48.32	51.21	53.13	56.01	57.93	60.81	62.72	65.59	67.51	70.37	72.28
0.0	*44.77*	47.66	49.58	52.47	54.39	57.27	59.19	62.07	63.98	66.85	68.77
-0.1	41.54	44.44	46.36	49.26	51.18	54.07	55.99	58.87	60.79	63.67	65.58
-0.2	38.65	41.55	43.48	46.38	48.31	51.20	53.12	56.01	57.93	60.81	62.73
-0.3	36.09	39.00	40.93	43.83	45.76	48.66	50.59	53.48	55.40	58.29	60.21
-0.4	33.87	36.78	38.72	41.62	43.56	46.46	48.39	51.28	53.21	56.10	58.03
-0.5	31.99	34.90	36.84	39.75	41.69	44.59	46.52	49.42	51.35	54.25	56.18
-0.6	30.44	33.36	35.30	38.21	40.15	43.06	45.00	47.90	49.83	52.73	54.66
-0.7	29.24	32.16	34.10	37.02	38.96	41.87	43.81	46.72	48.65	51.55	53.49

Table 3. Sensitivity of selling option value according on asymmetry and kurtosis (Own elaboration)

ε \ κ	3.0	3.3	3.5	3.8	4.0	4.3	4.5	4.8	5.0	5.3	5.5
0.7	59.38	59.65	59.82	60.08	60.26	60.52	60.70	60.97	61.14	61.41	61.58
0.6	58.28	58.54	58.72	58.98	59.15	59.42	59.59	59.86	60.03	60.30	60.47
0.5	57.19	57.45	57.63	57.89	58.06	58.33	58.50	58.77	58.94	59.20	59.38
0.4	56.12	56.38	56.56	56.82	56.99	57.26	57.43	57.69	57.87	58.13	58.31
0.3	55.07	55.33	55.50	55.76	55.94	56.20	56.38	56.64	56.81	57.07	57.25
0.2	54.03	54.29	54.47	54.73	54.90	55.16	55.34	55.60	55.77	56.03	56.21
0.1	53.01	53.27	53.44	53.71	53.88	54.14	54.31	54.57	54.75	55.01	55.18
0.0	*52.01*	52.27	52.44	52.70	52.87	53.13	53.31	53.57	53.74	54.00	54.18
-0.1	51.02	51.28	51.45	51.71	51.88	52.14	52.31	52.57	52.75	53.01	53.18
-0.2	50.04	50.30	50.47	50.73	50.90	51.16	51.34	51.60	51.77	52.03	52.20
-0.3	49.08	49.34	49.51	49.77	49.94	50.20	50.37	50.63	50.81	51.07	51.24
-0.4	48.13	48.39	48.56	48.82	48.99	49.25	49.42	49.68	49.86	50.12	50.29
-0.5	47.20	47.45	47.63	47.88	48.06	48.31	48.49	48.75	48.92	49.18	49.35
-0.6	46.27	46.53	46.70	46.96	47.13	47.39	47.56	47.82	48.00	48.25	48.43
-0.7	45.36	45.62	45.79	46.05	46.22	46.48	46.65	46.91	47.08	47.34	47.52

If the project offers an exclusionary strategy (differing-investing or selling), and we assume normality on the underlying behavior, it is clear that if predictions on t=0 actually occur, the option to be exercised on t=5 is the selling one. However, if the underlying asset does not follow a stochastic normal behavior, the selected strategy will depend on the impact of moments in value. From tables 2 and 3, we are able to build the following table where we present the decision to be making on each pair (ε; κ).

Table 4. Sensitivity of strangle strategy value according on asymmetry and kurtosis (Own elaboration)

ε \ κ	3.0	3.3	3.5	3.8	4.0	4.3	4.5	4.8	5.0	5.3	5.5
0.7	135.99	139.11	141.19	144.30	146.37	149.47	151.54	154.64	156.71	159.80	161.87
0.6	129.35	132.47	134.55	137.67	139.75	142.86	144.93	148.04	150.10	153.20	155.27
0.5	123.06	126.18	128.27	131.39	133.47	136.58	138.66	141.77	143.84	146.95	149.02
0.4	117.11	120.24	122.32	125.45	127.53	130.66	132.74	135.85	137.93	141.04	143.11
0.3	111.50	114.64	116.73	119.86	121.95	125.07	127.16	130.28	132.36	135.48	137.55
0.2	106.24	109.39	111.48	114.62	116.71	119.84	121.92	125.05	127.13	130.26	132.34
0.1	101.33	104.48	106.58	109.72	111.81	114.95	117.04	120.17	122.26	125.38	127.47
0.0	*96.77*	99.92	102.02	105.17	107.26	110.40	112.50	115.63	117.72	120.85	122.94
-0.1	92.56	95.71	97.81	100.96	103.06	106.21	108.30	111.44	113.54	116.67	118.76
-0.2	88.69	91.85	93.95	97.11	99.21	102.36	104.46	107.60	109.70	112.84	114.93
-0.3	85.17	88.33	90.44	93.60	95.71	98.86	100.96	104.11	106.21	109.35	111.45
-0.4	82.00	85.17	87.28	90.44	92.55	95.71	97.81	100.97	103.07	106.22	108.31
-0.5	79.18	82.36	84.47	87.63	89.74	92.91	95.01	98.17	100.27	103.43	105.53
-0.6	76.72	79.89	82.01	85.18	87.29	90.45	92.56	95.72	97.83	100.99	103.09
-0.7	74.60	77.78	79.89	83.07	85.18	88.35	90.46	93.63	95.74	98.90	101.00

Table 5. Sensitivity about selling or differing according on asymmetry and kurtosis (Own elaboration)

ε \ κ	3.0	3.3	3.5	3.8	4.0	4.3	4.5	4.8	5.0	5.3	5.5
0.7	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ
0.6	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ
0.5	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ
0.4	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ
0.3	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ
0.2	sell	differ	differ	differ	differ	differ	differ	differ	differ	differ	differ
0.1	sell	sell	sell	differ	differ	differ	differ	differ	differ	differ	differ
0	sell	sell	sell	sell	differ	differ	differ	differ	differ	differ	differ
-0.1	sell	sell	sell	sell	sell	differ	differ	differ	differ	differ	differ
-0.2	sell	sell	sell	sell	sell	differ	differ	differ	differ	differ	differ
-0.3	sell	sell	sell	sell	sell	sell	differ	differ	differ	differ	differ
-0.4	sell	sell	sell	sell	sell	sell	sell	differ	differ	differ	differ
-0.5	sell	sell	sell	sell	sell	sell	sell	differ	differ	differ	differ
-0.6	sell	sell	sell	sell	sell	sell	sell	differ	differ	differ	differ
-0.7	sell	sell	sell	sell	sell	sell	sell	sell	differ	differ	differ

4. Conclusion

Financial and real assets returns' distributions hardly ever adjust to the classic normal behavior having asymmetry and weight on the extremes. This characteristic makes valuation a complex problem which influences the strategic decision making in companies, such as decisions to expand, to differ, or to sell a project.

This paper has proposed valuing strategic assets using real option theory making adjustments that allow us to abandon the assumption of normal returns in continuous time. This technique permits the expansion's coefficient to depend also on the higher moments (asymmetry and kurtosis), either the original distribution or the approached one. Therefore, we obtained theoretical solutions to asset valuations that would have been difficult to solve.

Thus, this paper proposes to valuate this type of entrepreneurships using real options theory making adjustments that allow us to abandon the assumption of normal returns in continuous time. This technique permits the expansion's coefficient to depend also on the higher moments, either the original distribution or the approach one. Therefore, we presented theoretical results to valuation difficulties that would have been impossible to approach.

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Annex: Data published on Yahoo Finance. Call option for Facebook, Inc. Expiring on January, 15th, 2016. (Own elaboration)

Strike	Contract	Last	Bid	Ask
13.00	FB160115C00013000	62.75	62.45	63.05
15.00	FB160115C00015000	62.20	60.45	61.10
18.00	FB160115C00018000	57.00	57.50	58.15
20.00	FB160115C00020000	56.45	55.50	56.15

23.00	FB160115C00023000	52.65	52.55	53.20
25.00	FB160115C00025000	51.90	50.60	51.25
30.00	FB160115C00030000	46.55	45.70	46.35
33.00	FB160115C00033000	45.63	42.75	43.40
35.00	FB160115C00035000	41.00	40.80	41.45
38.00	FB160115C00038000	40.60	37.90	38.60
40.00	FB160115C00040000	36.20	36.00	36.70
43.00	FB160115C00043000	34.00	33.20	33.85
45.00	FB160115C00045000	32.35	31.35	31.90
47.00	FB160115C00047000	29.10	29.55	30.05
50.00	FB160115C00050000	27.23	26.85	27.35
52.50	FB160115C00052500	25.00	24.70	25.00
55.00	FB160115C00055000	22.95	22.65	22.95
57.50	FB160115C00057500	20.90	20.65	21.05
60.00	FB160115C00060000	19.00	18.70	19.10
62.50	FB160115C00062500	17.20	16.85	17.25
65.00	FB160115C00065000	15.30	15.10	15.45
67.50	FB160115C00067500	13.45	13.55	13.70
70.00	FB160115C00070000	12.20	12.00	12.15
72.50	FB160115C00072500	10.78	10.55	10.75
75.00	FB160115C00075000	9.45	9.45	9.50
77.50	FB160115C00077500	8.23	8.10	8.25
80.00	FB160115C00080000	7.11	7.05	7.15
82.50	FB160115C00082500	6.25	6.00	6.20
85.00	FB160115C00085000	5.25	5.20	5.30
87.50	FB160115C00087500	4.65	4.40	4.55
90.00	FB160115C00090000	3.90	3.75	3.90
95.00	FB160115C00095000	2.77	2.70	2.80
100.00	FB160115C00100000	1.98	1.93	2.02
105.00	FB160115C00105000	1.44	1.43	1.47
110.00	FB160115C00110000	1.13	0.96	1.06
115.00	FB160115C00115000	0.77	0.68	0.77
120.00	FB160115C00120000	0.51	0.47	0.55
125.00	FB160115C00125000	0.39	0.35	0.40
130.00	FB160115C00130000	0.29	0.25	0.30
135.00	FB160115C00135000	0.24	0.18	0.22
140.00	FB160115C00140000	0.17	0.15	0.17
145.00	FB160115C00145000	0.26	0.10	0.15
150.00	FB160115C00150000	0.10	0.06	0.13
155.00	FB160115C00155000	0.13	0.06	0.11